**NTU SSS Economics HE2001**  
**Tutorial 7 (Welfare Measures)**

1) Tom has a quasi-linear utility function . Illustrate in a diagram how for any price increase of good , compensating variation equals to equivalent variation.

*Note: for the shape of the utility function, you can take the indifference curves to be similar to that which we drew in the lecture.*

Blue solid line: old budget line, Black solid line: new budget line. (orange spots: old and new optimal consumption bundles)

For quasi-linear utility, the indifference curves are just (vertical) parallel shifts of each other.

For CV, we want to shift the Black solid line upwards to the Black dotted line such that same utility as before. (The amount shifted is equal the vertical shift of the indifference curves because of quasi-linear utility.)

For EV, we want to shift the Blue solid line downwards to the Blue dotted line such that utility becomes the same as new utility. (The amount shifted is also equal the vertical shift of the indifference curves)

(The red arrows show that the vertical shifts of the black and blue budget lines and the indifference curves are the same)

2) Bernice’s preferences can be represented by ,where is pairs of earrings and is dollars to spend on other things. She faces prices and her income is 12.

(a) Draw in a graph, her budget constraint and several indifference curves, illustrating the optimal bundle.

*Her optimal bundle is (4,4).   
Indifference curves reflect perfect complements, so she has to consume where   
Substituting this in the budget constraint gives us this solution.*

6

12

Earrings

Dollars on other things

(4,4)

(b) The price of a pair of earrings rises to $3 and Bernice’s income stays the same. On the same graph, draw her new budget constraint, illustrating the optimal point.

6

12

Earrings

Dollars on other things

4

Her optimal bundle is again where . Solving this using the same method, we get .

(4,4)

(3,3)

(c) What bundle would Bernice choose if she faced the original prices and had just enough income to reach the new indifference curve in (b)? Draw a budget line which passes through this bundle at the original prices. What is the level of income Bernice has at this new budget line?

*If she just has enough income to reach the new indifference curve, it must be tangent which implies it is at (3,3). The new budget line is drawn in red and reflects a budget of .*

(d) What is the equivalent variation (EV) of the price increase in earrings from $2 to $3 ?  
Remember that EV is the maximum income that at the original prices, when removed, just gives the consumer the same final utility level.

*One needs to remove $3 from the initial budget at original prices in order to get to the least income which gives us the same final utility level at the new prices. Hence EV is $3.*

***Alternatively, we can use the indirect utility function.*** *Demand will be where .  
Substituting this into the utility function, indirect utility is .*

*EV is where . Substituting values, we get , or .*

(e) What bundle would Bernice choose if she faced the new prices and had just enough income to reach the original indifference curve in (a)? Draw a budget line which passes through this bundle at the new prices. What is the level of income Bernice has at this budget line?

*If she just has enough income to reach the old indifference curve, it must be tangent which implies it is at (4,4). The new budget line is drawn in green and reflects a budget of .*

(f) What is the compensating variation (CV) of the price increase in earrings from $2 to $3 ?  
Remember that CV is the least income that at the new prices, when given, just restores the consumer’s original utility level.

*One needs to add $4 to the initial budget at new prices in order to get to the same utility level at the original prices. Hence CV is $4.*

*Using the indirect utility method:  
CV is where ,*

*Notice that CV is different from EV.*

*Note that in this question, the derivation of CV and EV is simpler because given that our utility involves perfect complements, the only two consumption bundles which are relevant are the final and original utility bundles. In general, as shown in your lecture notes, there are extra points of tangency that you will have to calculate when you shift prices.*

3) Steve consumes earplugs and other goods. His utility function is increasing in both his consumption of earplugs and other goods . Suppose that Steve has a sufficiently large wealth .

1. Suppose and that Steve can buy units of earplugs at total cost .
2. Write out Steve’s utility of consuming units of earplugs and spending the rest on other goods.
3. Suppose that the earplug costs , write out Steve’s utility of consuming units of earplugs and spending the rest on other goods.
4. Show that the reservation price for the earplug is .   
   (Hint: look at the definition of the reservation price.)

*Note that this implies that if earplugs were perfectly divisible, the reservation price function .*

1. The reservation price is the maximum price he is willing to pay for the nth unit.

This occurs when: Simplifying this gives the answer.

Intuitively, equating the utilities in 1ai) and 1aii) gives us the price which makes him just indifferent between purchasing (ii) and not purchasing the nth unit (i).

The price which makes him just indifferent between purchasing (ii) and not purchasing the nth unit (i) is the reservation price of the unit because below the below that price, he will still want to purchase the nth unit, but above that price he will not want to purchase the nth unit.

This is because as the price increases, his utility of consuming a total of earplugs decreases.   
I.e. it is the maximum amount of money he is willing to pay for the nth good.

1. Solve Steve’s utility maximisation problem (subject to prices and wealth to get his inverse demand function for earplugs . Is it the same as the reservation price function?

Using MRS=price ratio,

So , which is the same as before.

1. In the lecture’s example, and such that the reservation price curve is . What do you think the additional term captures?

can be interpreted as a factor which converts utility changes to monetary values. For this utility function, each unit of utility is always worth units of which is worth units of money.  
When and and the other good is implicitly money, then utility is directly convertible into monetary units.

4) Lolita, an intelligent and charming Holstein cow, consumes only two goods, cow feed (made of ground corn and oats) and hay. Her preferences are represented by the utility function , where is her consumption of cow feed and is her consumption of hay. Lolita has been instructed in the mysteries of budgets and optimization and always maximizes her utility subject to her budget constraint. Lolita has an income of that she is allowed to spend as she wishes on cow feed and hay. The price of hay is always $1, and the price of cow feed will be denoted by , where .

1. Write Lolita’s inverse demand function for cow feed.

Using MRS=price ratio,

1. If the price of cow feed is and her income is , how much hay does Lolita choose?
2. Plug these numbers into her utility function to find out the utility level that she enjoys at this price and this income.

Note that this is just the indirect utility function.

1. Suppose that Lolita’s daily income is $3 and that the price of feed is $0.50. What bundle does she buy? What bundle would she buy if the price of cow feed rose to $1?

At

At

1. Suppose that Lolita could pay a fee to avoid the price increase. What is the largest fee Lolita be willing to pay to avoid having the price of cow feed rise to $1? Is this EV or CV?

Substituting the values from d)

At . At

Hence, she faces a loss of units of utility.

At price , the maximum amount of money we can take away from her such that she has a utility of at least 3 is . *(If is fixed at , consumption of cow feed always remains fixed at 0.5, hence the change in utility when we remove money is just the drop in consumption of hay .)*

She would thus be willing to pay to avoid having the price of cow feed rise to $1.   
Note that this amount is exactly equal to equivalent variation.

Alternatively, we can use the indirect utility definition of EV to get this amount.

Taking

Then EV is where

or .

1. Suppose that the price of cow feed rose to $1. How much extra money would you have to pay Lolita to make her as well-off as she was at the old prices? Is this EV or CV?

1/8. This amount is known as the compensating variation, which is equal to EV in this question.

It can be calculated from

They are equal because she has quasi-linear utility. You can also calculate this directly via the same reasoning as above.

1. What is the change in consumer surplus of the price increase of cow feed from $0.5 to $1?

1/8. When utility is quasi-linear, considering a price decrease, change in CS=|CV|=|EV|, so the value should be +1/8.

You can also verify this by calculating the consumer surplus from the inverse demand curve for cow feed.

Initial consumer surplus =

Final consumer surplus

**Sample Questions (No solutions will be provided for these)**

1) There are two types of fruits grown for consumption in Fruitland, A and B, with initial prices and respectively. Due to climate change, a recent drought caused the price of A to increase to . The price of B, remains the same.

Assume that a typical consumer in Fruitland has cobb douglas utility , and an income of , where and is the consumption of A and B respectively

1. Illustrate graphically in separate diagrams, (i) the compensating variation and (ii) the equivalent variation of the price increase. **(12 marks)**
2. Suppose , and . Calculate the exact value of the compensating variation. **(10 marks)**
3. Will equivalent variation be equal to compensating variation in this case? Explain. *(Hint: you don’t need to calculate EV)***(5 marks)**

2) Bob, who has wealth of is deciding where to go for tea-time. There are 3 possible options, i) , ii) , iii) , and he can choose only 1 of the options.

If he chooses an option, its value is 1. If he does not choose the option, its value is 0.  
For example, if he chooses , then and so on.

His utility depends on which item he chooses and his leftover holdings of cash as follows: , where .

1. State the definition of quasi-linear utility. Is Bob’s utility quasi-linear? **(4 marks)**
2. Solve for Bob’s reservation prices for option , option and option . **(6 marks)**